Metacognition within mathematics: A new and practical multi-method approach

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The importance of metacognition for student learning has been widely acknowledged (Biggs, 1987, Birenbaum, 1996, Brown and De Loache, 1983, Pintrich, and De Groot, 1990, Schoenfeld, 1987, Wilson and Wing Jan, in press). But the practicalities associated with teaching for metacognition and monitoring metacognition are not clear. This paper is concerned about the assessability of metacognition within mathematics. It is asserted that unless metacognition can be assessed then it will exist as a theoretically sound construct but never be considered a viable part of the mathematics curriculum.

Introduction

This paper reports on the results of a PhD pilot study that has been conducted with upper primary students in the curriculum domain of mathematics. It reports on what students said they did when they solved different types of mathematics problems. The pilot study highlighted key aspects of students' metacognitive thinking and raises important methodological questions about the validity of assessing of students' thinking in mathematics. A new, practical multi-method interview has been developed as a result of the pilot study to meet the methodological challenge of researching metacognition. It responds to questions of legitimacy and meets the needs of classroom researchers.

Project Aims

The study aimed to identify strategies for assessing metacognition within mathematics. Three subsequent questions were used to focus the study:

1. Which assessment strategies are most effective?
2. What do these assessment strategies reveal about the nature of metacognition?
3. What is the relationship between metacognition and task type?

Rationale

For over two decades many curriculum documents have promoted the importance of metacognition for improving the educational outcomes of students (The Australian National Statement on Mathematics, Australian Educational Council, 1991, Baird and Northfield, Eds, 1992, Biggs, 1987, Ministry of Education, 1989, Stacey, 1990, Wittrock, Ed, 1986). But there is much confusion about the meaning of metacognition. The term 'metacognition' has not been clearly defined, little teacher reference material about metacognition has been written and even less assessment material is available to monitor metacognition. With little teacher support, an 'overcrowded curriculum' (Pigdon and Woolley, Eds, 1992) and a move to test the basics, it is argued that metacognition will never be considered achievable as a curricula goal.

Clarke (1988) argues that assessment should monitor the development of the attributes and capabilities that we value. This paper asserts that metacognition should be assessed within mathematics to increase the likelihood of it being taught. The development of 'teacher friendly' assessment materials of metacognition would improve the likelihood of metacognition being seen as important, viable and basic to student and curriculum development.

Metacognition and mathematics

Research in the field of metacognition has mostly focussed on gifted or disabled learners, often in the area of literacy. Much work has been conducted at tertiary levels or with adults. A number of researchers have asserted the centrality of metacognition for effective mathematical thinking, especially in relation to problem solving (Schoenfeld, 1990). In research studies investigating mathematics and metacognitive decision making, Goos (1993, 1994, 1995) claims an important link: 'Metacognitive processes are considered to be an important factor influencing problem solving performance.' (1993:1) Schoenfeld, 1987, Venezky and Bregar, (1990) agree that the ability to monitor one's own learning increases the effectiveness of problem solving.

Stacey claims that: 'Good problem solvers tend to show more meta-cognitive knowledge. ie an awareness of the processes of learning and mathematics.'(1990:6) Research in the field of mathematics and metacognition has reported that students having difficulties in mathematics do not use a range of cognitive or metacognitive strategies Cardelle-Elewar (1992) and Munro (1993). It is no longer considered appropriate for students to just practise and memorise a collection of unrelated facts. For students to become self-directed problem solvers, teachers need to teach students to monitor and regulate their own learning. These suggestions of an association between metacognition and able mathematics problem solvers, strengthens the case for investigating the assessment of metacognition and has important implications for teaching and assessment practices.

Defining metacognition and its functions

Despite a growing interest in the concept of metacognition over the last twenty years and attempts to define reflective and metacognitive thinking, a lack of clarity has existed (Brown, 1987 and Munro, 1993). The terms used in this paper are defined below after a brief review of the literature.

The literature associated with metacognition dates back to Dewey in 1933. Although he didn't use the word metacognition, he spoke of 'reflective self awareness',-the importance of active, persistent and careful consideration of beliefs and knowledge. Flavell (1976) was the first to use the term 'metacognition', which refers to the individual's awareness, consideration and control of his or her own cognitive processes and strategies. Since then a variety of meanings given to the term.

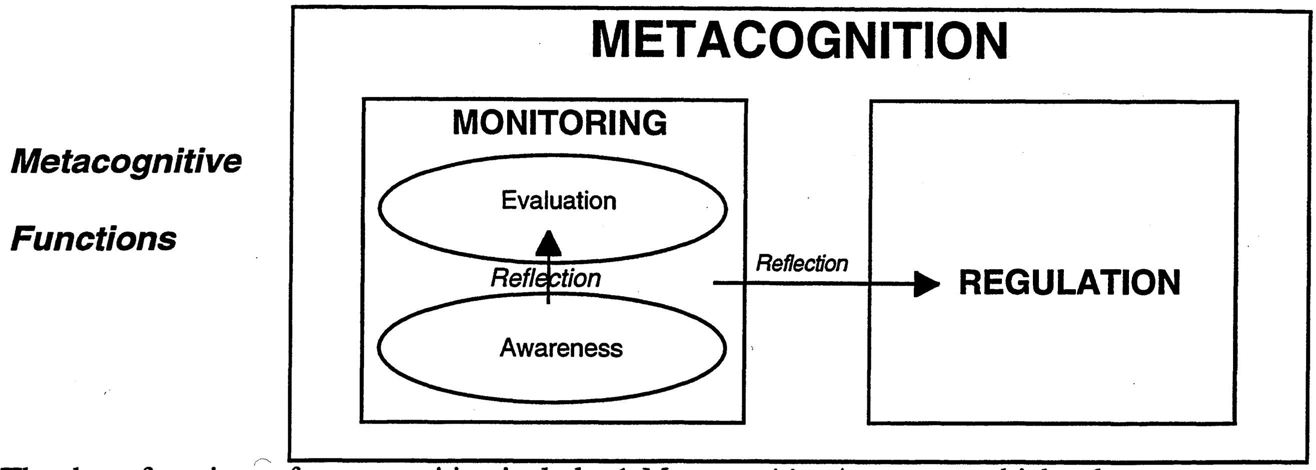
Perry (1989) summarised theorists' attempts to classify and describe metacognition. She refers to Kontos (1983) who describes knowing and conscious control of cognitive processes, and Brown, Bransford, Ferrara, and Campione, (1983) who state that knowledge about cognition and self regulation of cognition are equally important twin aspects of metacognition. Brown defined metacognition simply as referring to 'one's knowledge and control of one's own cognitive system.' (1987:67). Schraw and Dennison (1994) include a reflective component in their definition of metacognition: 'Metacognition refers to the ability to reflect upon, understand and control one's learning.' (1994:460)

An understanding of the term metacognition is made difficult because it is used to refer to two distinct areas of research: knowledge about cognition and regulation of cognition, Garofalo and Lester (1984), Brown (1987) and Schoenfeld (1990). The problem of definition has implications for methodology because the definition provides the parameters for research and a means to analyse the results of the investigation.

My synthesis of the preceding literature overview takes the form of the following definition: Metacognition refers to the *awareness individuals have of their own thinking and their ability to evaluate and regulate their own thinking.*

Figure 1 represents metacognition as it is employed in this paper. It is used as a framework and may be helpful for teaching practitioners. The model shows that there are three functions of metacognition which include: Awareness, Evaluation and Regulation of one's own thinking. Awareness and Evaluation are components of the thinking activity classified as Monitoring. Reflection is the mediating process whereby Awareness may become Evaluation and Evaluation may be transformed into Regulation of the thinking processes.

Figure 1. Model of Metacognition



The three functions of metacognition include: 1.*Metacognitive Awareness* which relates to an individual's awareness of where they are in the learning process, their knowledge about personal learning strategies and what needs to be done. 2.Metacognitive *Evaluation* which refers to judgements made regarding one's thinking capacities and limitations as these are employed in a particular situation or as self-attributes. For example, individuals could be making a judgement on the effectiveness of their thinking and strategy choice. 3.*Metacognitive Regulation* which occurs when individuals modify their thinking. They make use of their metacognitive skills to control their knowledge and thinking. They reflect on their knowledge about self and strategies (how and why they may use particular strategies). Metacognitive Regulation may include the ability to plan, self-correct, set goals and the effective use of one's own cognitive resources.

It is acknowledged that metacognition occurs within a context (such as classrooms) and that other aspects related to learning, such as students' prior knowledge, abilities, preferred ways of learning, expectations (Biggs, 1993) and volition (Corno, 1993) affect the nature of learning and thinking. These have not been discounted as important because they have not been represented on the model (Figure 1).

Methodology

The methodology used in this study was naturalistic. The data collection was primarily qualitative, deemed more suitable for aspects of life which are not directly observable. The methods used in this study were chosen after consideration of the following:

- questions of legitimacy in assessing metacognition,
- the effects of the various data collection techniques on metacognitive behavior and the
- suitability of each method when used with children.

Questions of legitimacy

There are many objections raised about the legitimacy of researching and assessing metacognition. These objections are explored and addressed in this paper. For example, Metacognition cannot be assessed because:

1. An agreement on a definition has not been achieved.
2. Researchers can only assess the consequences of metacognition.
3. The validity of self-reporting (a technique often associated with metacognitive research) is questionable (Nisbett and Wilson, 1977, Nuthall and Alton-Lee, 1995).

Garofalo and Lester (1984) suggest there are at least three reasons why the link between metacognition and mathematics performance has not been studied systematically. Firstly, they claim, mental activity is difficult to observe and analyse. If, as some psychologists suggest, people have no direct access to their mental processes, self-reports should be considered highly questionable. Increased linguistic abilities are sometimes given as a reason as to why student metacognitive awareness and control is reported to increase with age. Thorpe and Satterly (1990) share a concern for the adequacy of self-reporting measures of metacognition suggesting that results may reflect language development rather than metacognitive change.

Secondly, Garofalo and Lester (1984) argue that when self-reports are used while subjects perform a task this may stimulate metacognition rather than provide data on cognition and metacognition. Clarke (1992) also alerts us to the influence assessment has on student mathematical behavior and performance. He reports that assessment distorts the behavior it was intended to monitor.

Thirdly, Garofalo and Lester (1984) assert that as long as metacognition remains ill-defined, research in the area will attract little attention. Although the literature search results would agree with the lack of shared meaning for the term metacognition, the amount of research conducted in the field suggests that the issue has created intrigue and engaged many researchers, particularly since 1984. Nevertheless, the difficulties in assessment of metacognition in mathematics continue to exist and to thwart educators and researchers.

Methods overview

Pilot Study Sample

Questionnaires and interviews were trialled with two groups of grade six children (total n=15) in Melbourne. One group were from an outer suburban school (A, n=8). 5 of these 8 students were interviewed immediately after the questionnaire implementation. A further 7 questionnaires were given to an inner suburban cohort of children.

Questionnaires and Interviews

Likert scales and open ended questions were chosen as part of the questionnaire and interview process. Because of the difficulty of misinterpretation of questions, pretesting of the questions was conducted with a similar sample of children at another site before the pilot study. The distribution of responses from school A was similar to that of school B. The results of the teacher questionnaire have not been reported in detail in this paper.

Self-reporting

Students were asked to solve two problems in the questionnaire and then to record what they did using Likert scales (see inventory items below). The interviewed students were asked to solve another problem and report on their thinking using another set of the same Likert scales. Students were not asked to talk aloud (as in some studies, for example, Lesgold, Lajoie, Logan, and Eggan, 1990) as they solved the problems because of the possibility that verbalisation could effect, rather than accurately monitor metacognition (Clarke, 1992).

Types of mathematics tasks

Two mathematics problems were used as a basis for students' reflections on their own metacognitive thinking. The problems were non-routine and challenging because it is recognised that some students do not have to reflect to solve many classroom mathematics problems (Fortunato et al, 1991). Some mathematics tasks do not demand the use of metacognitive functions therefore careful selection was required. The tasks needed to provide an effective basis for reflection in the inventory sections of the questionnaire. Further discussion about the connection between various mathematics task types and metacognition is beyond the scope of this paper.

Inventory

The questionnaire was based around an inventory of metacognitive behaviors presented on Likert scales. The inventory items were constructed after a thorough examination of the literature and based on my definition of metacognition. This technique has been used in other studies of metacognition (Fortunato, Hecht, Kehr Tittle, and Alvarez 1991, Stacey, 1990, Goos, 1995, and Grubaugh and Speaker, Jr. 1991-2). In the search of education literature, no inventories were found for assessing the metacognition of children at the grade six level in mathematics. Most have been completed with students at tertiary or secondary levels.

The mathematics questionnaires used by Fortunato et al (1991) and Stacey (1990) were influential in the design of the questionnaire. Both were conducted with secondary school students therefore they needed to be simplified for primary age children.

Students were asked to indicate whether they always, sometimes or never did the following things when they solved mathematics problems :

a. Think about what you already know
b. Ask yourself a question about the problem
c. Think about what the problem is asking you to do.
d. Make a plan to work it out
e. Try to remember if you have ever done a problem like this before
f. Guess the answer because it is like a problem you've done before
g. Think about what you will do next
h. Go back and check your work
i. Think about a different way to solve the problem
j. Change the way you are working

Some of these items have since been modified as a result of the pilot study.

Results

The results of the questionnaires and interviews are summarised here. The findings about the research methods are considered important because the effectiveness of the instruments for gathering data on metacognition raises questions for further studies on mathematics and metacognition. More detailed results are reported in Wilson (1997).

Results about metacognition within the context of mathematics

Subjects were grouped according to the similarity of their responses to particular items. Items represent metacognitive functions as shown in the brackets.

Most students said they *always* :

a. Thought about what they already know. (awareness)
c. Thought about what the problem was asking them to do. (awareness)
d. Made a plan to work it out. (regulation)
e. Tried to remember if they have ever done a problem like this before. (awareness)

People often suggest that student reports of their behaviors merely reflect their teachers instructions and values. In this study there was little correspondence between what the interviewed teacher said she did when instructing the class in mathematics and what the students reported. Interestingly, on items where students demonstrated a range of responses, the teacher said she *never* instructed her students to ask questions about the problem (item b), that she *sometimes* asked them to think about what they would do next (item g) and that she *always* asked students to go back and check their work (item h).

Results about research methods

Student Questionnaire: When children are asked to recall decisions they make during mathematics problem solving they often do not report metacognitive thinking. Including a problem in the questionnaire provided a context for students to reflect on. Because the use of metacognition varies between contexts, in other studies, students could be asked to report on a problem just completed as well as what they usually do when they solve mathematics problems (retrospective reflection).

Student Interviews: The interviews provided opportunity for students to elaborate on responses and to follow up discrepancies between questionnaire responses on different tasks. The interviews also provided an opportunity to observe students as they solved another problem. Many student responses highlighted difficulties in reporting on one's own thinking. For example, when asked about their mathematics problem solving strategies, many students replied similarly: “Don't know”, “It came automatic”, “Just known before” and “Did it in my head”. These sort of responses are common and frustrating for teachers who are trying to gauge student understandings and misconceptions. Sometimes they reveal a lack of metacognition and at other times demonstrate that the problem solving process has become automatic.

Some student comments offered insights into their thinking. For example, the following quote suggests that regulation of thinking may be related to the perceived importance of the task. Tests seemed to provide the motivation for encouraging action. This response followed a question about student self-assessment in mathematics.

> "Yes, occasionally I think about how I could have done it better. ... I think about my answers and if it's good and correct. If I could do it a better way. Try different ways. Even try them, sometimes. If I think it's a better way to do mathematics questions and you get better marks I'd probably write it out better-if it's like a test. If it's like normal classroom mathematics, I always try hard but I put more effort into a test."

Discussion and Implications

The findings of this small project raise many questions about the use of and assessment of metacognition in mathematics. One of the reasons why metacognition has presented such a methodological challenging research topic is because of the difficulties in clarifying what metacognition actually is. I believe the logistics of assessing metacognition in classrooms are partly based on this problem. The following key findings about assessing metacognition are made. Some suggestions for reducing the difficulties associated with assessing and researching metacognition. These have implications for teachers and teacher educators.

Key finding 1: *Existing definitions of metacognition are insufficiently precise and difficult to interpret in operational terms.* A workable model of metacognitive behavior has been provided in this paper. A framework is important for teachers interested in changing their mathematics curriculum delivery (Borkowski and Muthukrishna, 1992) and assessment techniques.

Key finding 2: *Self-reporting appears to be a valid method for researching metacognition when used in conjunction with other reliable data collection methods.* Corroborating evidence should be provided, for example through teacher observations. In classrooms a simple inventory of metacognitive behaviors, such as the one outlined in this paper, or the multi-method approach outlined below could be used. Tasks which require an achievable level of challenge are required as a context for reflections on metacognition. Where there is a mismatch between students' self-assessment and the teacher's assessment, further validation could be required.

A new multi-method approach

A new technique which is based upon the design outlined in this paper and draws on the strengths of many commonly used assessment strategies has been pretested as a result of this pilot study. The new technique has been given the term 'multi-method interview'. It includes aspects of the pilot questionnaire: verbal Likert scale, self-assessment, observation, clinical interview activity, think aloud technique (where chosen by the participant), and audio recording. This technique requires minimum preparation and little time to administer. It could be easily applied to many mathematics contexts and to a range of educational settings.

The technique would make use of the revised items on the Likert scales but the procedure would be implemented in a more creative and 'hands on' way. Subjects would be asked to have a go at solving a mathematics problem. Each metacognitive behavior item would be listed individually on a playing card (Clarke, 1989) for subjects to sort and sequence according to how they solved the problem. Cards listing cognitive behaviors and blank cards would also be provided. The resultant sequence of metacognitive behaviors suggested by subjects could be used to confirm or refute the postulated sequence of metacognitive functions (awareness, evaluation and regulation). Cards which did not apply would be discarded for later discussion about students usual metacognitive behavior when solving mathematics problems. Each card could be used to elicit information about how often students might do these things when solving mathematics problems, for example, on all problems or a few problems. Discussion could raise issues about the connection between different mathematics tasks and use of metacognitive functions.

Conclusion

It has been argued that assessment practices in mathematics must include assessment of student metacognitive thinking. Otherwise assessment procedures are capable of denying students the opportunity to use and develop metacognitive thinking which is so crucial in mathematics. If assessment of students' metacognitive thinking is not integral to mathematics teaching and learning, the improvement of student metacognition cannot be assumed: only a basic mathematics curriculum for our students can be expected.

To be attractive to practising classroom teachers, an assessment strategy for metacognition, such as the new multi-method approach outlined, must be easily integrated into mathematics instruction.

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